

Attendance Problems. Identify the hypothesis and conclusion of each conditional.

1. A mapping that is a reflection is a type of transformation.

2. The quotient of two negative numbers is positive.

3. Determine if the conditional “If x is a number, then $|x| > 0$ ” is true. If false, give a counterexample.

I can apply the law of detachment and the law of syllogism in logical reasoning.

Vocabulary: Deductive reasoning.

Common Core

- ✓ CC.9-12.G.CO.9 Prove theorems about lines and angles.
- ✓ CC.9-12.G.CO.10 Prove theorems about triangles.
- ✓ CC.9-12.G.CO.11 Prove theorems about parallelograms.
- ✓ CC.9-12.G.SRT.4 Prove theorems about triangles.

“I don’t have my homework today because. . .” Is your teacher going to be convinced? Will it make a difference whether you say that the dog ate your homework or whether you bring in a note from the doctor? Imagine that your friend says , “I know that shape is a square because it has four right angles.” Did your friend tell you enough to convince you?

Many jobs depend on your ability to convince other people that your ideas are correct. For instance, a defense lawyer must be able to form logical arguments to persuade the jury or judge that his client is innocent.

Trial of the Century

The musical group *Apple Core* has accused your math teacher, Mr. Gripentrog of stealing is newest CD, *Rotten Gala*. According to the police, someone stole the song from the BigCD store last Saturday at sometime between 6:00 pm and 7:00 pm. Because your class is so well known for only reaching conclusions when sufficient evidence is presented, the judge has made you the jury! You are responsible for whether or not there is enough evidence to convict Mr. Gripentrog.



4. Carefully listen to the evidence that is presented. As each statement is read, decide on the following:

- *Does the statement convince you?*
- *Why or why not? What could be changed/added to the statement to make it more convincing?*

Statement	Convinced? (Y/N)	Why or why not? What could be changed/added to make it more convincing
1		
2		
3		

Statement	Convinced? (Y/N)	Why or why not? What could be changed/added to make it more convincing
4		
5		
6		
7		
8		

You are at home when the phone rings. It is a good friend of yours who says, “Hey, your last name is Marston. Any chance you have a grandmother named Molly Marston who was REALLY wealthy? “Check out today’s paper.” You glance at the front page:

Family Fortune Unclaimed

City officials are amazed that the county’s largest family fortune may go unclaimed. Molly “Ol’ Granny” Marston died earlier this week and it appears that she was survived by no living relatives. According to her last will and testament, “Upon my death, my entire fortune is to

be divided among my children and grandchildren.” Family members have until noon tomorrow to come forward with a written statement giving evidence that they are related to Ms. Marston or the money will be turned over to the city.

You are amazed--Molly is your grandmother, so your friend is right! However, you may not be able to collect your inheritance unless you can convince city officials that you are a relative. You rush to your attic where you keep a trunk full of family memorabilia.



5. You find several items that you think might be important in an old trunk in the attic. With your table group, underline/highlight which of the items listed below that will help prove the Ol’ Molly was your grandmother.

☐ **Family Portrait** -- a photo showing three young children. On the back you see the date 1968.

☐ **Newspaper Clipping** -- an article from 1972 titled “Triplets Make Music History.” The first sentence captures your eye. “Jake, Judy, and Jeremiah Marston, all 8 years, were the first triplets ever to perform a six handed piano piece at Carnegie Hall.”

☐ **Jake Marston’s Birth Certificate** -- showing that Jake was born in 1964, and identifying his parents as Phillip and Molly Marston.

☐ **Your Learners Permit** -- signed by your father, Jeremiah Marston.

☐ **Wilbert Marston’s passport** -- issued when Wilbert was 15.

6. Your table will now write a statement that will convince the city official (played by Mr. Gripenot) that Ol' Molly was your grandmother. Be sure to support any claims that you make with appropriate evidence. Sometimes it pays to be convincing!

7. The day before Gerardo returned from a two-week trip, he wondered if he left his plants inside the apartment or outside on his deck. He knows these facts:

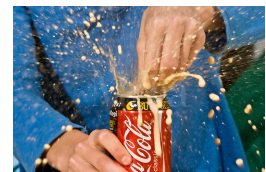
- ☛ If the plants are indoors, he must water them at least once a week or they will die.
- ☛ If leaves his plants outdoors and it rains, he does not have to water them. Otherwise he must water them at least once a week or they will die.
- ☛ It has not rained in his town for 2 weeks.

When Gerardo returns, will his plants be dead? Explain how you know.

8. What is deductive reasoning?

Video Example 1. Is the conclusion a result of inductive or deductive reasoning?

A. There is a myth that tapping the side of a carbonated beverage will prevent it from foaming over. However, if you were to observe a carbonated beverage that has been shaken, tapped, and then opened, you would see that this myth is false.



B. There is a myth that if a person goes swimming less than one hour after eating they will likely get a cramp and drown. However, studies has shown that there has never been a death from drowning or a near drowning attributed to eating before swimming. Therefore, the myth cannot be true.



1 **Media Application**

Urban legends and modern myths spread quickly through the media. Many Web sites and television shows are dedicated to confirming or disproving such myths. Is each conclusion a result of inductive or deductive reasoning?

A There is a myth that toilets and sinks drain in opposite directions in the Southern and Northern Hemispheres. However, if you were to observe sinks draining in the two hemispheres, you would see that this myth is false. Since the conclusion is based on a pattern of observation, it is a result of inductive reasoning.



B There is a myth that you should not touch a baby bird that has fallen from its nest because the mother bird will disown the baby if she detects human scent. However, biologists have shown that birds cannot detect human scent. Therefore, the myth cannot be true. The conclusion is based on logical reasoning from scientific research. It is a result of deductive reasoning.



Example 1. Is the conclusion a result of inductive or deductive reasoning?

A. There is a myth that you can balance an egg on its end only on the spring equinox. A person was able to balance an egg on July 8, September 21, and December 19. Therefore this myth is false.



B. There is a myth that the Great Wall of China is the only man-made object visible from the Moon. The Great Wall is barely visible in photographs taken from 180 miles above Earth. The Moon is about 237,000 miles from Earth. Therefore, the myth cannot be true.



4. Guided Practice: There is a myth that an eelskin wallet will demagnetize credit cards because the skin of the electric eels used to make the wallet holds an electric charge. However, eelskin products are not made from electric eels. Therefore, the myth cannot be true. Is this conclusion a result of inductive or deductive reasoning?



In deductive reasoning, if the given facts are true and you apply the correct logic, then the conclusion must be true.

Law of Detachment

If $p \rightarrow q$ is a true statement and p is true, then q is true.

Video Example 2. Determine if the conjecture is valid by the Law of Detachment.

A. Given: If M is the midpoint of a line segment, then M bisects the segment. M is the midpoint of \overline{AB} .

Conjecture: $\overline{AM} \cong \overline{MB}$



B. Given: If you are absent more than 2 times, you will fail the class.

Ed failed the class.

Conjecture: Ed was absent two or more times.

2 Verifying Conjectures by Using the Law of Detachment

Determine if each conjecture is valid by the Law of Detachment.

A Given: If two segments are congruent, then they have the same length.
 $\overline{AB} \cong \overline{XY}$.

Conjecture: $AB = XY$

Identify the **hypothesis** and **conclusion** in the given conditional.

If **two segments are congruent**, then **they have the same length**.

The given statement $\overline{AB} \cong \overline{XY}$ matches the hypothesis of a true conditional. By the Law of Detachment $AB = XY$. The conjecture is valid.

B Given: If you are tardy 3 times, you must go to detention.
 Shea is in detention.

Conjecture: Shea was tardy at least 3 times.

Identify the **hypothesis** and **conclusion** in the given conditional.

If **you are tardy 3 times**, **you must go to detention**.

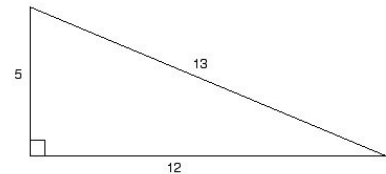
The given statement "**Shea is in detention**" matches the conclusion of a true conditional. But this does not mean the hypothesis is true. Shea could be in detention for another reason. The conjecture is not valid.

Example 2. Determine if the conjecture is valid by the Law of Detachment.

A. Given: If the side lengths of a triangle are 5 cm, 12 cm, and 13 cm, then the area of the triangle is 30 cm^2 .

The area of $\triangle PQR$ is 30 cm^2 .

Conjecture: The side lengths of $\triangle PQR$ are 5cm, 12 cm, and 13 cm.



B. Given: In the World Series, if a team wins four games, then the team wins the series. The Red Sox won four games in the 2004 World Series.

Conjecture: The Red Sox won the 2004 World Series.



5. Guided Practice: Determine if the conjecture is valid by the Law of Detachment. Given: If a student passes his classes, the student is eligible to play sports. Ramon passed his classes.

Law of Syllogism

If $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement.

Video Example 3. Determine if the conjecture is valid by the Law of Syllogism.

A. Given: If $90^\circ < m\angle A < 180^\circ$, then $\angle A$ is obtuse. If $\angle A$ is obtuse, then it is not an acute angle.

Conjecture: If $90^\circ < m\angle A < 180^\circ$, then it is not an acute angle.

B. Given: If a number is a multiple of 10, then it is a multiple of 5. If the last digit of a number is 0, then it is a multiple of 5.

Conjecture: If a number is a multiple of 10, then the last digit of the number is 0.

3 Verifying Conjectures by Using the Law of Syllogism

Determine if each conjecture is valid by the Law of Syllogism.

A Given: If $m\angle A < 90^\circ$, then $\angle A$ is acute. If $\angle A$ is acute, then it is not a right angle.

Conjecture: If $m\angle A < 90^\circ$, then it is not a right angle.

Let p , q , and r represent the following.

p : The measure of an angle is less than 90° .

q : The angle is acute.

r : The angle is not a right angle.

You are given that $p \rightarrow q$ and $q \rightarrow r$. Since q is the conclusion of the first conditional and the hypothesis of the second conditional, you can conclude that $p \rightarrow r$. The conjecture is valid by the Law of Syllogism.

Determine if each conjecture is valid by the Law of Syllogism.

B **Given:** If a number is divisible by 4, then it is divisible by 2.
If a number is even, then it is divisible by 2.

Conjecture: If a number is divisible by 4, then it is even.

Let x , y , and z represent the following.

x : A number is divisible by 4.

y : A number is divisible by 2.

z : A number is even.

You are given that $x \rightarrow y$ and $z \rightarrow y$. The Law of Syllogism cannot be used to draw a conclusion since y is the conclusion of both conditionals. Even though the conjecture $x \rightarrow z$ is true, the logic used to draw the conclusion is not valid.

Example 3. Determine if the conjecture is valid by the Law of Syllogism.

A. Given: If a figure is a kite, then it is a quadrilateral. If a figure is a quadrilateral, then it is a polygon.

Conjecture: If a figure is a kite, then it is a polygon.



B. Given: If a number is divisible by 2, then it is even. If a number is even, then it is an integer.

Conjecture: If a number is an integer, then it is divisible by 2.

6. Guided Practice: Determine if the conjecture is valid by the Law of Syllogism. Given: If an animal is a mammal, then it has hair. If an animal is a dog, then it is a mammal.

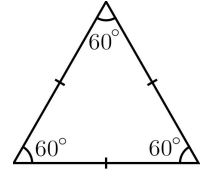


Video Example 4. Draw a conclusion from the given information.

A. If Ed runs 10 miles on Monday, then he will not run the next day. If Ed does not run on Tuesday, then he will run 10 miles on Wednesday. Ed ran 10 miles on Monday.



B. If a triangle is equilateral, then it has three 60° angles. If a triangle has three 60° angles, then it is not an obtuse triangle. \triangle is equilateral.



4 Applying the Laws of Deductive Reasoning

Draw a conclusion from the given information.

A Given: If a team wins 10 games, then they play in the finals. If a team plays in the finals, then they travel to Boston. The Ravens won 10 games.

Conclusion: The Ravens will travel to Boston.

B Given: If two angles form a linear pair, then they are adjacent. If two angles are adjacent, then they share a side. $\angle 1$ and $\angle 2$ form a linear pair.

Conclusion: $\angle 1$ and $\angle 2$ share a side.

Example 4. Draw a conclusion from the given information.

A. Given: If $2y = 4$, then $z = -1$. If $x + 3 = 12$, then $2y = 4$. $x + 3 = 12$.

Conjecture: $z = -1$.

B. If the sum of the measures of two angles is 180° , then the angles are supplementary. If two angles are supplementary, they are not angles of a triangle. $m\angle A = 135^\circ$, and $m\angle B = 45^\circ$.

2-3 Using Deductive Reasoning to Verify Conjectures: (pp 91) 9-13, 16, 18-22, 26, 28